BC Calc Vector 3 Name: Key 2025 Block: Seat:

1. (No Calc) The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 2$, $y(t) = \frac{2}{3}t^3$.

   (a) Find the magnitude of the velocity vector at $t = 2$.

$$\vec{s} = \langle t^2 - 2, \tfrac{2}{3}t^3 \rangle$$
$$\vec{v} = \langle 2t, 2t^2 \rangle$$
$$\vec{a} = \langle 2, 4t \rangle \Big|_{t=2} = \langle 2, 8 \rangle$$
$$\|\vec{a}\| = \sqrt{2^2 + 8^2} = 4\sqrt{5}$$

   (b) Set up an integral expression to find the total distance traveled by the particle from $t = 0$ to $t = 4$.

$$\int_0^4 \|\vec{v}\|\, dt = \int_0^4 \|\langle 2t, 2t^2 \rangle\|\, dt = \int_0^4 \sqrt{4t^2 + 4t^4}\, dt$$

($\simeq$ 46.061 or 46.062 BTW)

   (c) Find $\frac{dy}{dx}$ as a function of x.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(\frac{2}{3}t^3\right)}{\frac{d}{dt}\left(t^2 - 2\right)} = \frac{2t^2}{2t} = t$$

Since $x = t^2 - 2$
$t = \sqrt{x+2}$ so $\frac{dy}{dt} = t = \sqrt{x+2}$

   (d) At what time t is the particle on the y-axis? Find the acceleration vector at this time.

on y axis when x componet is 0
$t^2 - 2 = 0$ when $t = \sqrt{2}$

$$\vec{a} = \langle 2, 4t \rangle \Big|_{t=\sqrt{2}} = \langle 2\sqrt{2}, 4\sqrt{2} \rangle$$

2. (No Calc) An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with the ; velocity vector $v(t) = \langle (t+1)^{-1}, 2t \rangle$. At time $t = 1$, the object is at $(\ln 2, 4)$.

(a) Find the position vector.

Method 1

$\vec{s} = \int \langle \frac{1}{t+1}, 2t \rangle \, dt$

$= \langle \ln|t+1| + C_1 , t^2 + C_2 \rangle$

Since $\ln(1+1) + C_1 = \ln 2$, $C_1 = 0$

further, $1 + C_2 = 4$ so $C_2 = 4$

Hence $\vec{s} = \langle \ln|t+1| , t^2 + 3 \rangle$

Method 2

$\vec{s}(t) = \vec{s}(1) + \int_1^t \vec{v} \, dt$

$= \langle \ln 2, 4 \rangle + \int_1^t \langle \frac{1}{t+1}, 2t \rangle \, dt$

$= \langle \ln 2, 4 \rangle + \langle \ln|t+1|, t^2 \rangle \Big|_1^t$

$= \langle \ln 2, 4 \rangle + \langle \ln|t+1|, t^2 \rangle - \langle \ln 2, 1 \rangle$

$= \langle \ln|t+1|, t^2 + 3 \rangle$

(b) What is the speed of the particle when $t = 1$.

$$\text{speed} = \|\vec{v}\| = \sqrt{\left(\frac{1}{1+1}\right)^2 + [2(1)]^2}$$

$$= \sqrt{\frac{1}{4} + 4} = \frac{\sqrt{17}}{2}$$

(c) Write an equation for the line tangent to the curve when $t = 1$.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{\frac{1}{t+1}}\Bigg|_{t=1} = \frac{2}{\frac{1}{2}} = 4$$

Since @ $(\ln 2, 4)$ at $t=1$, tangent line is
$y - 4 = 4(x - \ln 2)$

(d) At what time $t \geq 0$ does the line tangent to the particle at $(x(t), y(t))$ have a slope of 12?

$2t(t+1) \geq 12$

$t^2 + t - 6 \geq 0$

$(t+3)(t-2) \geq 0$

$t = -3$ and $t = 2$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{\frac{1}{t+1}}\Big|_{t=2} = \frac{4}{\frac{1}{3}} = 12$ ✓

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{\frac{1}{t+1}}\Big|_{t=-3} = \frac{-6}{-\frac{1}{2}} = 12$ ✓

(e) Write an expression that represents how far has the particle travelled from time $t = 0$ to $t = 1$.

$$\int_0^1 \|\vec{v}(t)\| \, dt = \int_0^1 \sqrt{\left(\frac{1}{t+1}\right)^2 + (2t)^2} \, dt$$

$(\approx \underline{1.305}2)$

3. (Calc OK) A particle moving along a curve in the xy-plane has position $(x(t), y(t))$, with $x(t) = 2t + 3\sin t$ and $y(t) = t^2 + 2\cos t$, where $0 \le t \le 10$.
Find the velocity vector at the time when the particle's vertical position is $y = 7$.

$\vec{v}(t) = \langle 2 + 3\cos t,\ 2t - 2\sin t \rangle$

find t when $y = 7$

$t^2 + 2\cos t = 7$ at $t = \boxed{A} = 2.9964952$

(store this number no rounding yet!)

$\vec{v}(A) = \langle -0.968,\ 5.703 \rangle$

or $\langle -0.968,\ 5.704 \rangle$

4. (Calc OK) A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{dt} = 1 + \sin(t^3)$. The derivative $\frac{dy}{dt}$ is not explicitly given. For any time t, $t \ge 0$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $t + 3$. Find the acceleration vector of the object at time $t = 2$.

Since $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ we have $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (t+3)(1+\sin t^3)$

so $t > 0$

$\vec{a}(t) = \left\langle \frac{d}{dt}\left(\frac{dx}{dt}\right),\ \frac{d}{dt}\left(\frac{dy}{dt}\right) \right\rangle$

$\vec{a}(t) = \langle 3t^2 \cos t^3,\ (t+3)(3t^2\cos t^3) + (1 + \sin t^3)$

$\vec{a}(2) = \langle 12\cos 8,\ 60\cos 8 + 1 + \sin 8 \rangle$

$\vec{a}(2) = \langle -1.746,\ -\underline{6.740}6 \rangle$

Cde isn't really needed here

typo

5. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{\cancel{dy}\ dt} = \cos(e^t)$ and $\frac{dy}{dt} = \sin(e^t)$ for $0 \le t \le 2$. At time $t = 1$, the object is at the point $(3, 2)$.

(a) Find the equation of the tangent line to the curve at the point where t = 1.

$$\left.\frac{dy}{dx}\right|_{t=1} = \frac{\sin e^1}{\cos e^1} = \tan e$$

$$y - 2 = \tan e\,(x-3)$$

(b) Find the speed of the object at t = 1.

$$\|\vec{v}(t)\| = \sqrt{\cos^2(e^t) + \sin^2(e^t)} = \sqrt{1} = 1$$

same speed for all t

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 2$.

$$\int_0^2 dt = t\Big|_0^2 = 2 - 0 = 2$$

(d) Find the position of the object at time $t = 2$.

$$x(2) = x(1) + \int_1^2 \cos(e^t)\,dt = 3 + (-0.10425) = 2.895743347$$

$$y(2) = y(1) + \int_1^2 \sin e^t\,dt = 2 + (-0.324022) = 1.675977975$$

position : $(\underline{2.895}7, \underline{1.675}9)$

6. A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\dfrac{dx}{dt} = \sin(t^3 - t)$ and $\dfrac{dy}{dt} = \cos(t^3 - t)$. At time $t = 3$, the particle is at the point $(1, 4)$.

(a) Find the acceleration vector for the particle at $t = 3$.

$$\vec{v}(t) = \langle \sin(t^3 - t), \cos(t^3 - t) \rangle$$

$$\vec{a}(t) = \langle (3t^2 - 1)\cos(t^3 - t), -(3t^2 - 1)\sin(t^3 - t) \rangle$$

$$\vec{a}(3) = \langle 26 \cos 24, -26 \sin 24 \rangle$$

$$\text{about } \langle \underline{11.0286}, \underline{23.545} \rangle$$

(b) Find the equation of the tangent line to the curve at the point where $t = 3$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \left.\frac{\cos(t^3 - t)}{\sin(t^3 - t)}\right|_{t=3} = \frac{\cos 26}{\sin 26} = \cot 26$$

$$(\approx 0.84835)$$

$$y - 4 = \cot 26 (x - 1)$$

(c) Find the magnitude of the velocity vector at $t = 3$.

$$\| \vec{v}(t) \| = \sqrt{\sin^2 26 + \cos^2 26} = 1$$

(d) Find the position of the particle at time $t = 2$.

$$x = 1 + \int_3^2 \sin(t^3 - t)\, dt \approx 0.932$$

$$y = 4 + \int_3^2 \cos(t^3 - t)\, dt \approx 4.002$$

7. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dy}{dt} = 2 + \sin(e^t)$. The derivative $\frac{dx}{dt}$ is not explicitly given. At $t = 3$, the object is at the point $(4, 5)$.

(a) Find the y-coordinate of the position at time $t = 1$.

$$y = 5 + \int_3^1 2 + \sin(e^t)\, dt \approx 1.268 \text{ or } 1.269$$

$$\frac{dy}{dx} = \frac{-9}{5}$$

(b) At time $t = 3$, the value of $\frac{dy}{dx}$ is -1.8. Find the value of $\frac{dx}{dt}$ when $t = 3$.

Since $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ we have $\frac{dx}{dt} = \frac{dy}{dt} \cdot \left(\frac{dy}{dx}\right)^{-1}$

$= (2 + \sin e^t)\left(\frac{-5}{9}\right)$

$$\left.\frac{dx}{dt}\right|_{t=3} = \frac{-10}{9} - \frac{5}{9}\sin e^3 \approx 1.635 \text{ or } 1.636$$

(c) Find the speed of the object at time $t = 3$.

$$\|\vec{v}\| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \approx 3.368$$